Calculus for Team-Based Inquiry Learning

2026 Edition

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TBIL Community

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July 28, 2026

Website: Team-Based Inquiry Learning

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- *Active Calculus - single variable*
    - https://activecalculus.org/ACS.html
    - CC BY-SA 4.0

TBIL Resource Library. This book is a part of the TBIL Resource Library, which includes a growing collection of course materials for courses aligned with the TBIL pedagogy.

This material is based upon work supported by the National Science Foundation under Award No. 2011807. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

For Instructors

Team-Based Inquiry Learning. The book is not written as a traditional textbook. Instead, it is an *activity book* designed for use in a classroom using the Team-Based Inquiry Learning (TBIL) pedagogy. TBIL is an active learning pedagogy combining the philosophy of inquiry-based learning with the structure of Team-Based Learning.

A TBIL course is broken down into modules, which correspond to the chapters of this book. Each module starts with a **Readiness Assurance Process** to check understanding of and and solidify prerequisite knowledge (from other courses or previous modules) that will be needed in the upcoming module. The beginning of each chapter provides a list of these Readiness Assurance outcomes, along with study resources and exercise students can use to refresh themselves on this prerequisite knowledge. Readiness Assurance Tests that assess this knowledge are available as part of the TBIL Resource Libary. Join the TBIL community (see Community and Support) to gain access to these.

Within each module there is a varying amount of sections, one per learning outcome. Each section is designed to guide students into being able to demonstrate their understanding of that specific outcome. The learning outcome can be tested with a CheckIt Exercise, which are linked at the end of each section.

Within each section, students work in teams (typically during class) on the Class Activities, which are interspersed with definitions, theorems, remarks, examples, etc. as needed to guide the learning process. The activities begin in an exploratory way and then proceed to build concepts on the results of these explorations. The concepts are then practiced with decreasing amounts of scaffolding to build fluency. Finally, towards the end of the section we connect the concepts and extend them to new settings.

Each section/learning outcome is designed with an exercise, a specific set of tasks, that the students need to be able to solve to demonstrate their competency. Virtually limitless randomized versions of the exercise can be generated via CheckIt to build a problem bank and allow for reassessments.

Instructor Version. An instructor version of this text is available at `tbil.org`. The instructor version includes answers for the activities and facilitation notes for instructors interspersed through the book.

Community and Support. If you are adopting this text in your class, please fill out this short form so we can track usage, let you know about updates, etc.

Implementation of these materials is supported by a TBIL community of practice which offers both formal and informal professional devlopment opportunities. The best way to connect with us is on our online community chat.

These materials are a collaborative product of our TBIL community. Feedback and suggestions for improvements are most welcome, either through our GitHub Repository or our online community chat.

Video Resources. Videos are available at the end of each section. A complete playlist of videos aligned with this text is available in two parts on YouTube:

- Chapters LT through IN
- Chapters TI through PS

Contents

Chapter 1

Limits (LT)

Learning Outcomes

How do we measure "close-by" values?
By the end of this chapter, you should be able to...

1. Find limits from the graph of a function.
2. Infer the value of a limit based on nearby values of the function.
3. Compute limits of functions given algebraically, using proper limit properties.
4. Determine where a function is and is not continuous.
5. Determine limits of functions at infinity.
6. Determine limits of functions approaching vertical asymptotes.

Readiness Assurance.

Before beginning this chapter, you should be able to...

a Use function notation and evaluate functions

- Review: Khan Academy
- Practice:
  - Evaluate functions
  - Evaluate functions from their graphs
  - Function notation word problems

b Find the domain of a function

- Review: Khan Academy
- Practice: Determine the domain of functions

c Determine vertical asymptotes, horizontal asymptotes, and holes (removable discontinuities) of rational functions

- Review:
  - Discontinuities of rational functions
  - Finding horizonal asymptotes
- Practice:
  - Rational functions: zeros, asymptotes, and undefined points
  - Finding horizonal asymptotes

d Perform basic operations with polynomials

- Review:
  - Adding and subtracting polynomials
  - Multiplying polynomials
- Practice:
  - Add polynomials
  - Subtract polynomials
  - Multiply binomials by polynomials

e Factor quadratic expressions

- Review: Khan Academy
- Practice: Factoring quadratics intro

f Represent intervals using number lines, inequalities, and interval notation

- Review: Varsity Tutor

1.1 Limits Graphically (LT1)

Learning Outcomes

- Find limits from the graph of a function.

1.1.1 Activities

Activity 1.1.1 In Figure 1.1.2 the graph of a function is given, but something is wrong. The graphic card failed and one portion did not render properly. We can't see what is happening in the neighborhood of $x = 2$.

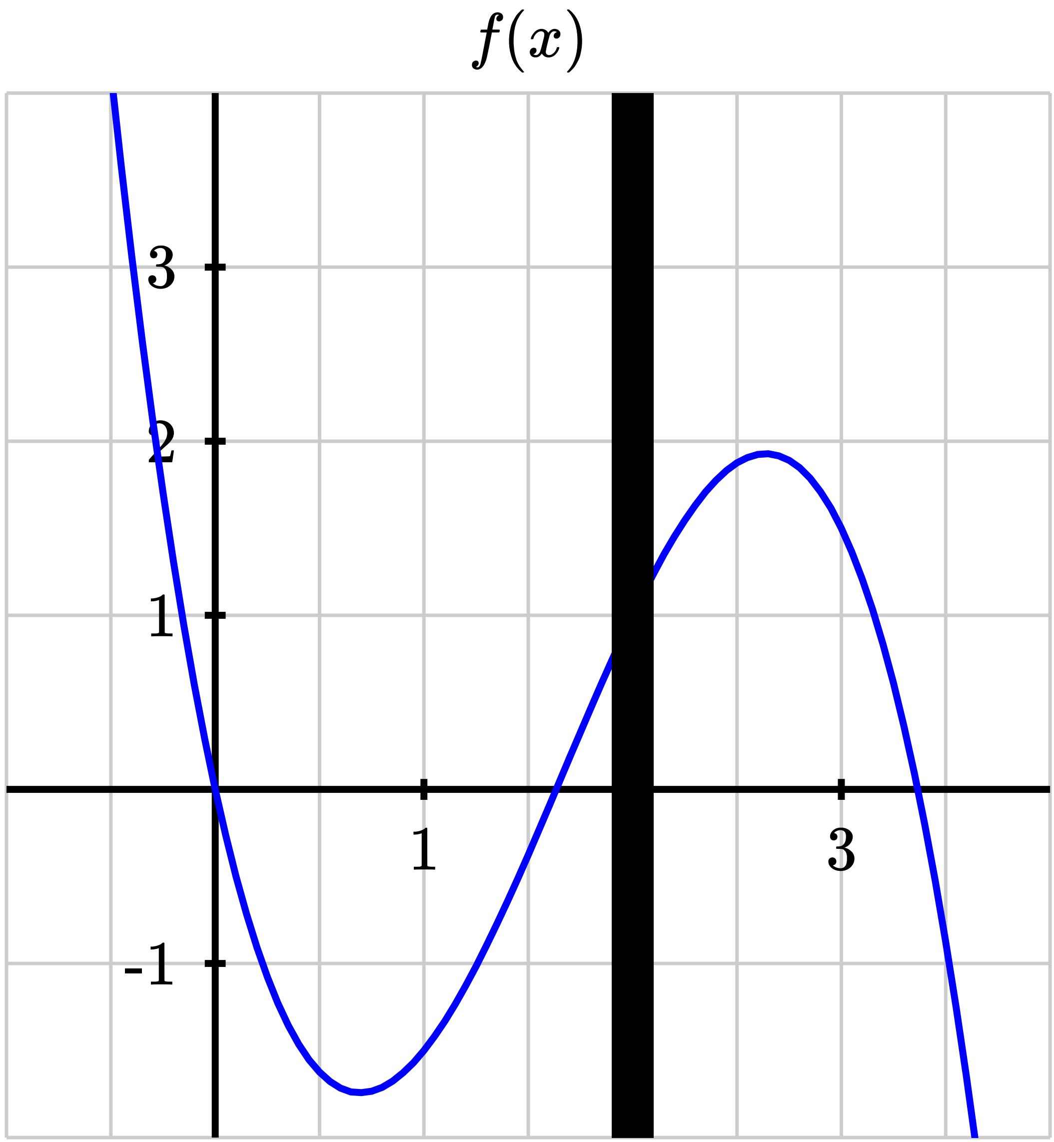

Figure 1.1.2 A graph of a function that has not been rendered properly.

(a) Imagine moving along the graph toward the missing portion from the left, so that you are climbing up and to the right toward the obscured area of the graph. What y-value are you approaching?

A. 0.5
B. 1
C. 1.5
D. 2
E. 2.5

(b) Think of the same process, but this time from the right. You're falling down and to the left this time as you come close to the missing portion. What y-value are you approaching?

A. 0.5
B. 1
C. 1.5
D. 2
E. 2.5

Activity 1.1.3 In Figure 1.1.4 the graphic card is working again and we can see more clearly what is happening in the neighborhood of $x = 2$.

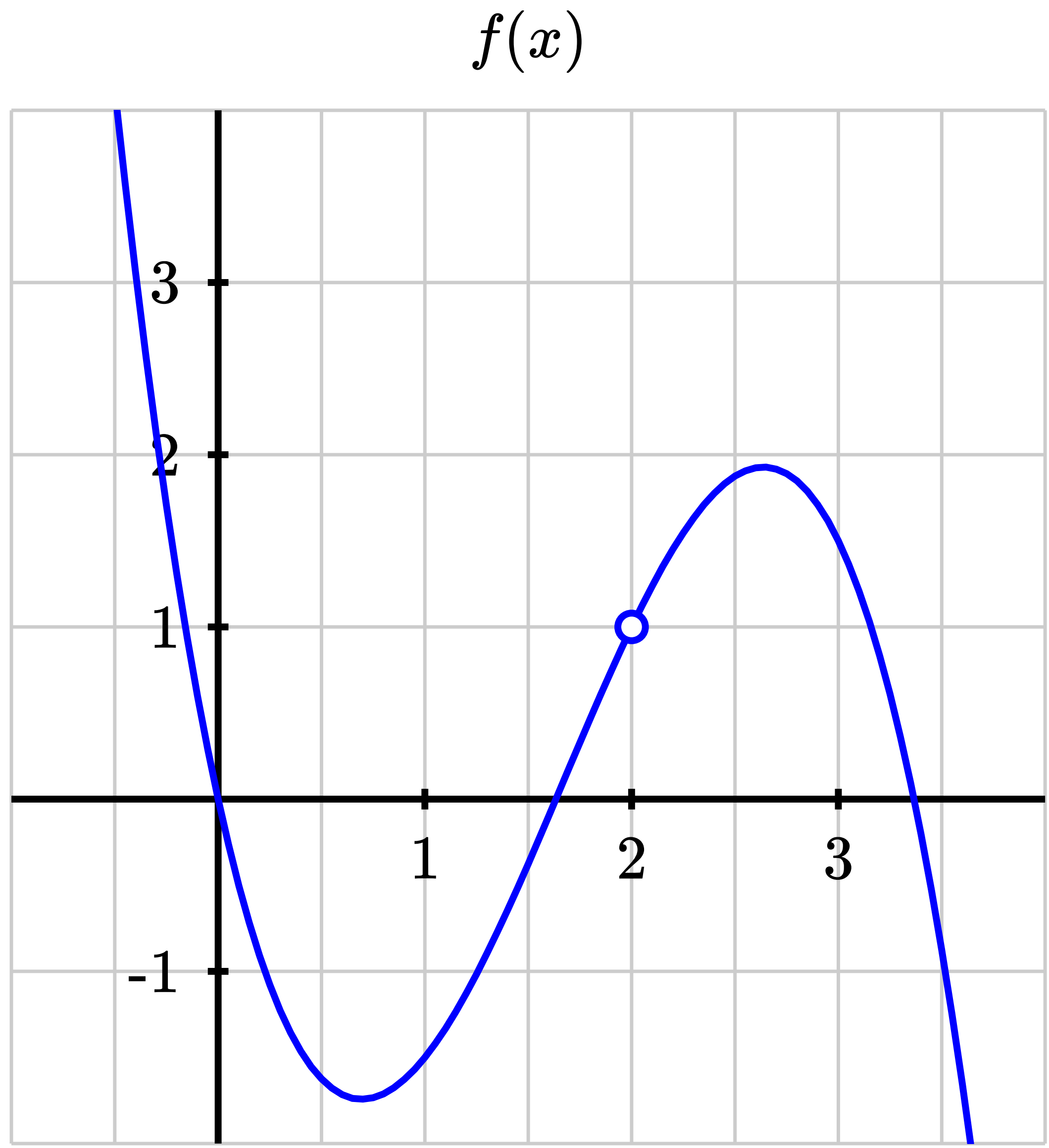

Figure 1.1.4 A graph of a function that has rendered properly

(a) What is the value of $f(2)$?

(b) What is the y-value that is approached as we move toward $x = 2$ from the left?

A. 0.5
B. 1
C. 1.5
D. 2
E. 2.5

(c) What is the y-value that is approached as we move toward $x = 2$ from the right?

A. 0.5
B. 1
C. 1.5
D. 2
E. 2.5

Remark 1.1.5 When studying functions in algebra, we often focused on the *value* of a function given a specific x-value. For instance, finding $f(2)$ for some function $f(x)$. In calculus, and here in Activity 1.1.1 and Activity 1.1.3, we have instead been exploring what is happening as we *approach* a certain value on a graph. This concept in mathematics is known as finding a limit.

Activity 1.1.6 Based on Activity 1.1.1 and Activity 1.1.3, write your first draft of the definition of a limit. What is important to include? (You can use concepts of limits from your daily life to motivate or define what a limit is.)

Definition 1.1.7 Given a function f, a fixed input $x = a$, and a real number L, we say that f **has limit** L **as** x **approaches** a, and write

$$\lim_{x \to a} f(x) = L$$

provided that we can make $f(x)$ as close to L as we like by taking x sufficiently close (but not equal) to a. If we cannot make $f(x)$ as close to a single value as we would like as x approaches a, then we say that f **does not have a limit as** x **approaches** a**.** $\Diamond$

Activity 1.1.8

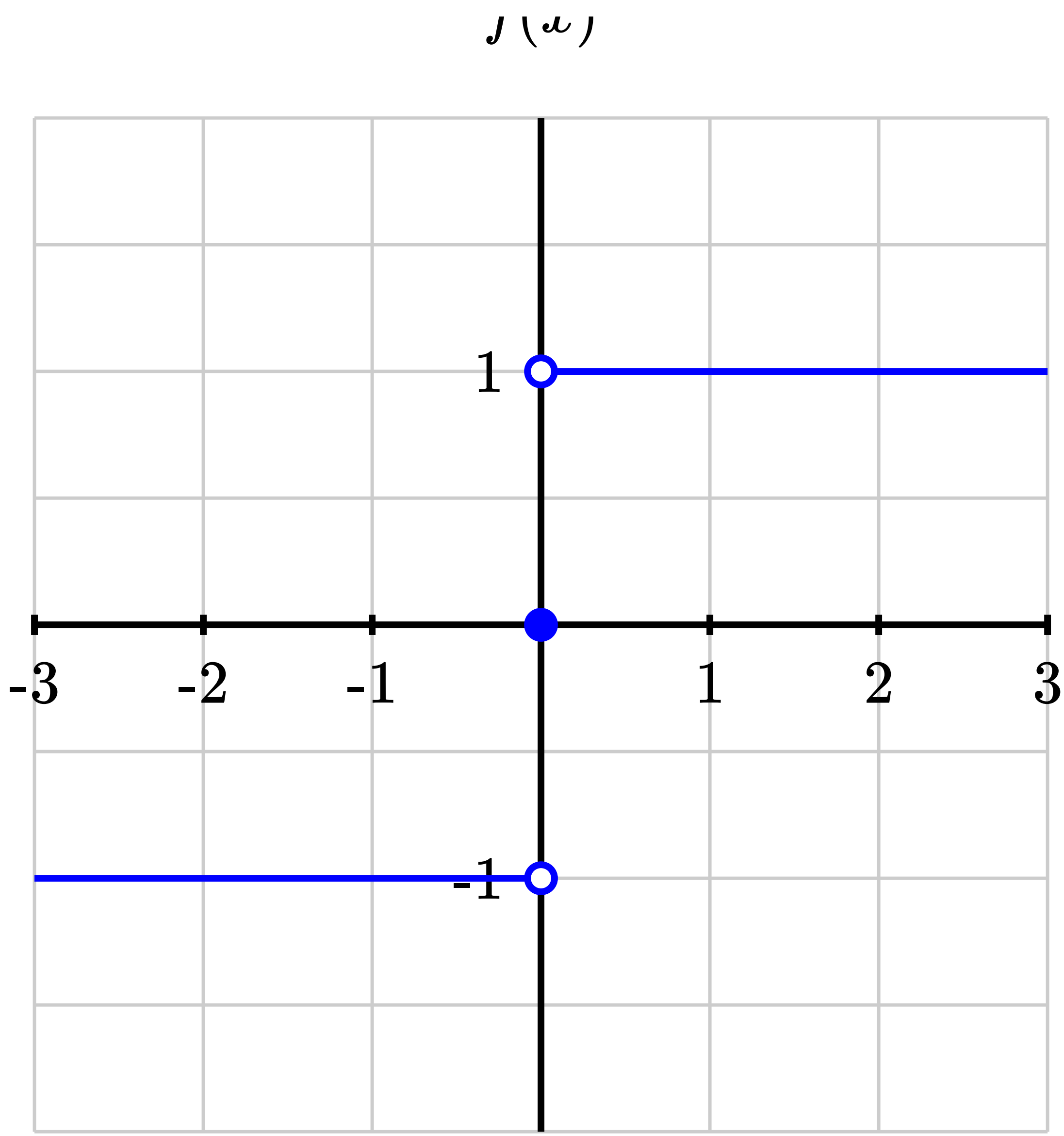

Figure 1.1.9 A piecewise-defined function

(a) What is the limit as x approaches 0 in Figure 1.1.9?

A. The limit is 1

B. The limit is -1

C. The limit is 0

D. The limit is not defined

Definition 1.1.10 We say that f *has limit* L_1 *as* x *approaches* a *from the left* and write

$$\lim_{x \to a^-} f(x) = L_1$$

provided that we can make the value of $f(x)$ as close to L_1 as we like by taking x sufficiently close to a while always having $x < a$. We call L_1 the left-hand limit of f as x approaches a.

Similarly, we say L_2 *is the right-hand limit of* f *as* x *approaches* a and write

$$\lim_{x \to a^+} f(x) = L_2$$

provided that we can make the value of $f(x)$ as close to L_2 as we like by taking x sufficiently close to a while always having $x > a$. $\diamondsuit$

Activity 1.1.11 Refer again to Figure 1.1.9 from Activity 1.1.8.

(a) Which of the following best matches the definition of right and left limits? (Note that DNE is short for "does not exist.")

A. The left limit is -1. The right limit is 1.

B. The left limit is 1. The right limit is -1.

C. The left limit DNE. The right limit is 1.

D. The left limit is -1. The right limit DNE.

E. The left limit DNE. The right limit DNE.

(b) What do you think the overall limit equals?

A. The limit is 1

B. The limit is -1

C. The limit is 0

D. The limit is not defined

Activity 1.1.12 Consider the following graph:

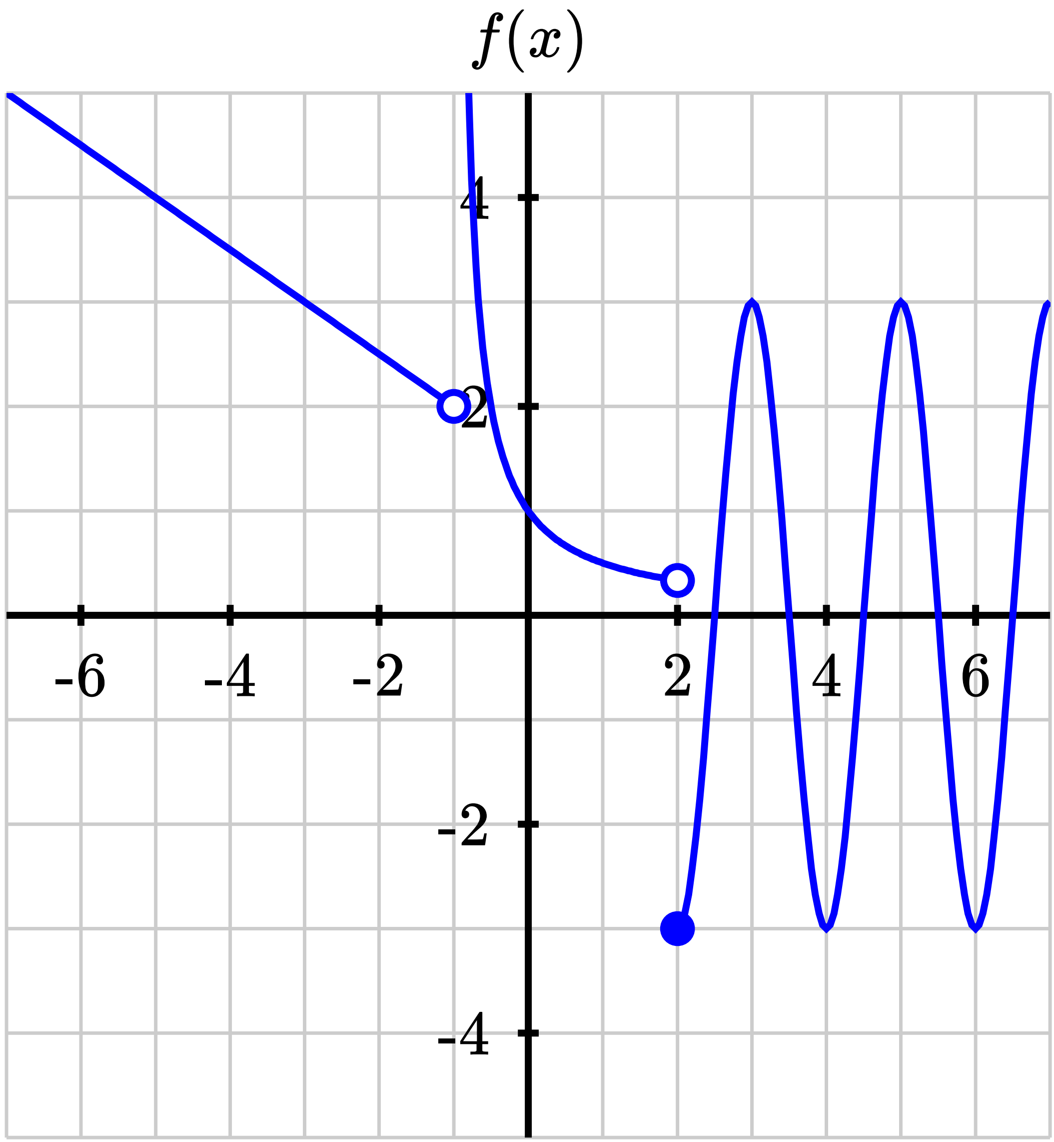

Figure 1.1.13 Another piecewise-defined function

(a) Find $\lim\limits_{x\to-3^-} f(x)$ and $\lim\limits_{x\to-3^+} f(x)$.

(b) Find $\lim\limits_{x\to-1^-} f(x)$ and $\lim\limits_{x\to-1^+} f(x)$.

(c) Find $\lim\limits_{x\to2^-} f(x)$ and $\lim\limits_{x\to2^+} f(x)$.

(d) Find $\lim\limits_{x\to4^-} f(x)$ and $\lim\limits_{x\to4^+} f(x)$.

(e) For which x-values does the *overall* limit exist? Select all. If the limit exists, find it. If it does not, explain why.

A. -3

B. -1

C. 2

D. 4

Activity 1.1.14 Sketch the graph of a function $f(x)$ that meets all of the following criteria. Be sure to scale your axes and label any important features of your graph.

1. $\lim_{x \to 5^-} f(x)$ is finite, but $\lim_{x \to 5^+} f(x)$ is infinite.
2. $\lim_{x \to -3} f(x) = -4$, but $f(-3) = 0$.
3. $\lim_{x \to -1^-} f(x) = -1$ but $\lim_{x \to -1^+} f(x) \neq -1$.

Theorem 1.1.15 *Suppose that:*

- *for an interval around $x = a$, we have that $f(x) \leq g(x) \leq h(x)$;*
- *the limit as x approaches a of $f(x)$ is equal to the same value L as the limit of $h(x)$, so $\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)$.*

Then, the limit of $g(x)$ as $x \to a$ is also L, so $\lim_{x \to a} g(x) = L$.

Activity 1.1.16 In this activity we will explore a mathematical theorem, the Squeeze Theorem (Theorem 1.1.15).

(a) The part of the theorem that starts with “Suppose…” forms the assumptions of the theorem, while the part of the theorem that starts with “Then…” is the conclusion of the theorem. What are the assumptions of the Squeeze Theorem? What is the conclusion?

(b) The assumptions of the Squeeze Theorem can be restated informally as “the function g is squeezed between the functions f and h around a.” Explain in your own words how the two assumptions result into a “squeezing effect.”

(c) Let’s see an example of the application of this theorem. First examine the following picture. Explain why, from the picture, it seems that both assumptions of the theorem hold.